measured temperature distribution was approximated by a parabolic fit, from which the temperature gradient was calculated (see the highest heating level data in Fig. 4). Use of the Fourier equation showed the heat-conduction contribution to the cooling of the wire at all stations, except at the centerline (where grad$T = 0$), to be about two orders of magnitude lower than the generated heat. Thus, the conduction influence can also be discarded.

Another possibility is that measurements with IR imaging systems of targets exhibiting high gradient temperature distributions are consistently lower than what they should be. The extent of this error is unknown (the manufacturer does not supply such information); therefore, an effort was made to estimate it by using the available experimental data. For this purpose, per-pixel calculations were made of

1) the electrical heat dissipation of the wire, based on current and resistance measured values;
2) the heat convected by the air, based on Eqs. (1) and (2);
3) the heat conducted along the wire, based on the measured temperature distribution and the Fourier law of heat conduction; and
4) the heat radiation, based on the measured temperature distribution and the Stephan-Boltzmann law.

Hence, the difference between item 1 and the sum of items 2–4 can be considered as an overall measurement error based on local energy balance. Figure 6 shows this error plotted against the measured temperature gradient at the location of each pixel for the highest heating level experiment. This finding is partially supported by the findings of Boylan et al.,[3] who showed that temperature measurements made with their IR system fell behind the actual temperature distribution once the temperature gradient of the target passed a certain limit (see their Fig. 12).

This conclusion does not diminish the merits of IR imaging systems for aerodynamic research. In fact, if the true error function was known, it would have been possible to adjust the experimental results to get the actual temperature distribution. Starting with the measured distribution, one can iteratively recover the true temperature distribution by adding the measured values distribution and the gradient dependent measurement error. Still to be answered is the question of whether this error is a function of the temperature rate of change of the target in the FOV, or in the IFOV, i.e., to what extent the measurement error is affected by the optics and the distance to the target, in addition to the actual temperature distribution.

In closing, it is suggested that the temperature distribution of the heated wire may be used to reconstruct the flow velocity distribution by using Eq. (2). This application of the method is illustrated in Fig. 7, where the only correction made was to subtract the heat conduction effects along the wire. The implementation of this method has to wait until the actual measurement errors of IR imaging systems are available, and their source understood.

Acknowledgments

This research was partially supported by NASA Langley Research Center Grant NAG1-735.

References

[1]Schlichting, H., *Boundary Layer Theory,* 7th ed., McGraw-Hill, New York, 1979, pp. 39, 241, 242.

[2]Morgan, V. T., "The Overall Convective Heat Transfer from Smooth Circular Cylinders," *Advances in Heat Transfer,* Vol. 11, Pergamon, New York, 1975.

[3]Boylan, D. E., Carver, D. B., Stallings, D. W., and Trimmer, L. L., "Measurement and Mapping of Aerodynamic Heating Using a Remote Infrared Scanning Camera in Continuous Flow Wind Tunnels," *Proceedings of the AIAA 10th Aerodynamic Testing Conference,* AIAA, New York, 1978, pp. 213–231.

Low-Order Panel Method for Internal Flows

K. Sudhakar* and G. R. Shevare†
Indian Institute of Technology, Bombay, India

Introduction

LOW-ORDER panel methods, if properly formulated, can yield accuracy levels comparable to higher order methods even for comparable panel densities.[1,2] Such low-order panel methods are currently popular as design tools in aeronautical industries, especially for external flows. The same ease and confidence is lacking, however, in applying low-order panel methods to internal flows or internal-external flows due to problems of accuracy.[3] A low-order panel method is proposed here that aims at reducing errors in the analysis of internal flows through complex three-dimensional ducts. A synthesized solution technique that can enhance solution accuracy of internal flow analysis or analyze internal-external flows in aircraft-intake combinations is also proposed.

The proposed low-order panel method uses constant source and doublet on flat panel approximations of the geometry to be analyzed. Such a scheme suffers from loss of accuracy as the higher order terms are neglected. This loss of accuracy may in some cases be avoided by using a large number of panels, but only at the cost of increased computations, negating the basic purpose of resorting to a low-order scheme. Since the most significant among the neglected terms in the low-order scheme is the gradient in doublet strength,[4] the accuracy can best be increased if gradients in the doublet distribution are somehow reduced. The proposed formulation aims at providing a handle to reduce gradients in the unknown doublet distribution a priori.

Analysis

The internal flowfield is considered as a perturbation field over an arbitrarily prescribed velocity V_o. The flow problem is solved by a formulation in which the doublet gradients become equal to this perturbation velocity. This is done by prescribing a source distribution of strength $\sigma = (v_n - V_o \cdot \boldsymbol{n})$ and solving for doublet strength μ by setting the external perturbation potential to zero:

$$\left. \iint_s (v_n - V_o \cdot \boldsymbol{n})(-1/r)\,\mathrm{d}s + \iint_s \mu \nabla(1/r) \cdot \boldsymbol{n}\,\mathrm{d}s = 0 \right|_p \tag{1}$$

Here $\boldsymbol{n}$ is the unit normal to the boundary s that points to the internal flow domain, v_n is the desired normal velocity boundary condition, and r is the distance from ds to any point p on the boundary but outside the flow domain. It can be shown that doublet strength μ here is equal to the perturbation potential ϕ on the boundary. The surface velocity along a direction $\boldsymbol{t}$, therefore, is

$$v_t = (\nabla\phi + V_o) \cdot \boldsymbol{t} = (\nabla\mu + V_o) \cdot \boldsymbol{t} \tag{2}$$

The doublet gradient in this formulation is equal to the surface perturbation velocity and may be reduced by a proper choice

Received May 23, 1990; revision received and accepted for publication July 28, 1990.

*Assistant Professor.
†Professor, Department of Aeronautical Engineering.

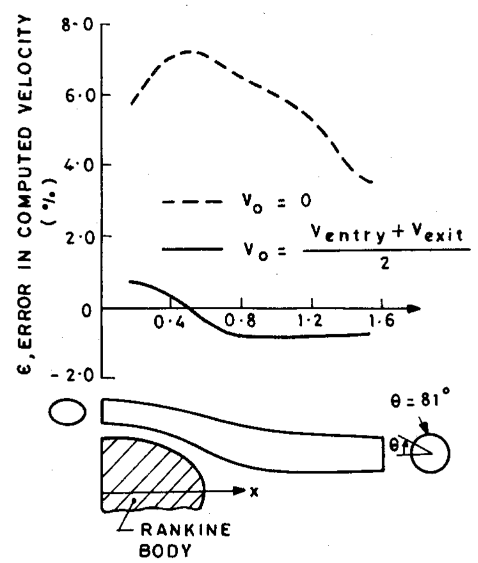

Fig. 1 Results for a duct with lateral offset.

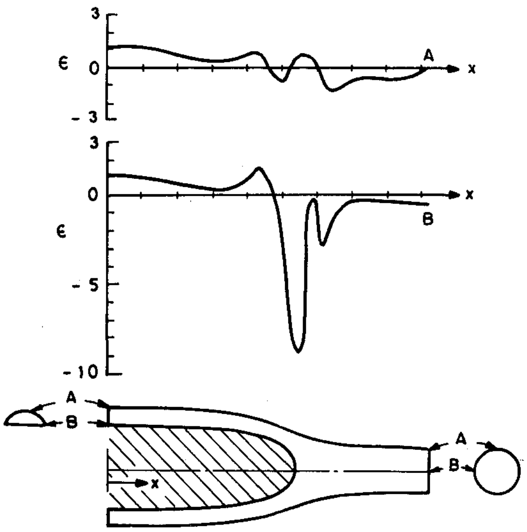

Fig. 2 Results for a bifurcated duct.

of V_o. For most duct flows, a good choice for V_o is the average of velocities at its entry and exit, though better choices can be made.

Results and Discussion

The effect of V_o on solution accuracy is shown in Fig. 1. The duct considered here is a stream tube traced in a flowfield generated by a source-sink of unit strength at a distance of ± 0.45 on the x axis in a unit onset flow (the flow past Rankine body). One end of the stream tube is a circle in the y-z plane with its center at (1.63, 0.2, 0) and a diameter of 0.2. The other end at $x = 0$ has an ellipse-like shape. The duct has an area ratio of 1.45, length-to-diameter ratio of 8.9, and lateral offset of 1.3 times the average diameter. The boundary conditions are such that v_n is zero on the duct surface and a nonuniform velocity profile, as defined by the flowfield, is at the duct ends. The exact solution is known in this case. The percentage error in velocity ϵ on a line along the duct length is shown in Fig. 1. When $V_o = 0$, the gradients in μ are equal to total surface velocity and the errors are large. When V_o is the average of velocities at the entry and exit of the duct, the gradients in doublet distribution and thus the errors in the obtained solutions are substantially reduced.

A bifurcated duct of area ratio 1.25 and length-to-diameter ratio 8.1 is generated by tracing a stream tube that is divided equally by the Rankine body in the above flowfield. This duct is analyzed with V_o as the average of velocity at its entry and exit. The percentage errors on two lines along the duct length with the smallest and largest errors on them are shown in Fig. 2. The 9% peak error in the solution occurs close to the stagnation point in the flow. But this is not alarming, since even a slight error in solution at the stagnation point will be unbounded when expressed as a percentage of the locally zero velocity.

Guo and Seddon[5] have tested an S-shaped air intake with a rectangular entry and circular exit. This duct has an area ratio of 1.338 and length-to-diameter ratio of 4.8. Discretized geometry of the duct is shown in Fig. 3. The lip shape outside the duct for this geometry is defined by Guo only for a short distance downstream of the highlight station. This was extended for the purpose of present computations. The solution inside the duct was insensitive to the shape of this extended portion outside. Results of present analysis compared reasonably well (Fig. 4) with the test data of Guo. The total pressure loss at the duct exit was measured by Guo as 11%. It may be noted that the computed C_p at this station is higher than Guo's test data by an equal amount, i.e., 11% of the total pressure at entry. The deviation between the two gradually vanishes toward the entry, as should be expected. This suggests that the deviation of present results from the test data of Guo is because of viscous effects.

Large number of ducts, for which analytical results could be generated, covering a wide range of area ratios L/d and lateral offsets, have been successfully analyzed by this low-order panel method. These studies have indicated that the error in solution at any point is strongly correlated with local doublet gradient for a given paneling. This information has been successfully exploited in proposing a synthesized solution tech-

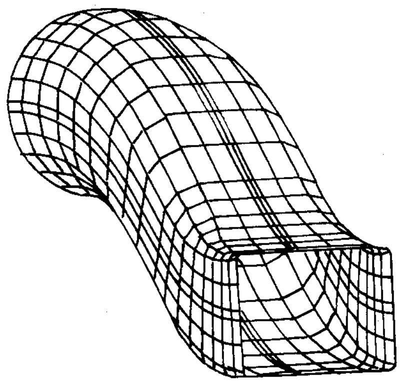

Fig. 3 Geometry of the S-shaped duct.

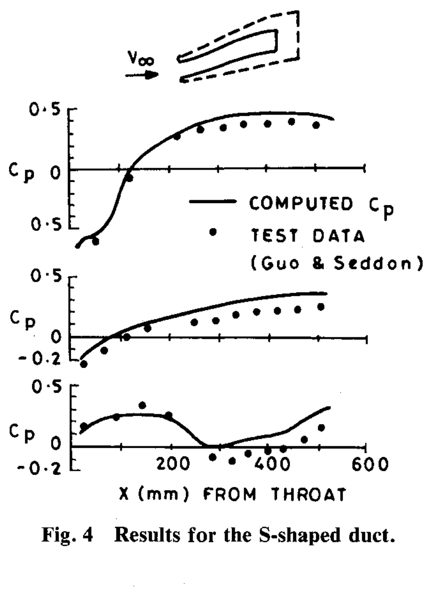

Fig. 4 Results for the S-shaped duct.

nique[4] that can enhance the accuracy of solution for a given discretization. The method involves generating two or more solutions to a problem for the same paneling. These multiple solutions are generated for different values of V_o. The choice of V_o for each analysis is based on the result of previous analysis and aims at refining the solution in the region where errors were large in the previous analysis. Such regions are easily spotted by scanning the doublet gradients in solutions, largest gradient indicating largest error. These multiple solutions can then be synthesized into one single solution of considerably increased accuracy by a simple procedure. For each panel, the solution corresponding to the one with the smallest doublet gradient is chosen. Generating multiple solutions is not computationally demanding since V_0 affects only the right side of the linear algebraic system set up for the problem. Once the matrix is set up and inverted for the first solution, every additional solution requires only a post multiplication. The additional time required to invert a matrix, as against obtaining one solution, will depend on whether a direct or iterative method is employed for finding the solution.

The method may be used for external flow analysis by setting the $V_0 = V_\infty$, whereby it coincides with the low-order panel method of Maskew.[1] In the case of internal-external flows such as aircraft-intake analysis, a synthesis of a minimum of two solutions, one for $V_o = V_\infty$ and another for $V_o = V_d$, where V_d is the average velocity expected in the intake duct, can be used.

Conclusions

The proposed low-order panel method can analyze flow in complex three-dimensional ducts, external flows, and internal-external flows.

References

[1]Maskew, B., "Prediction of Subsonic Aerodynamic Characteristics: A Case for Low-order Panel Methods," *Journal of Aircraft*, Vol. 19, No. 2, 1982, pp. 157–163.

[2]Miranda, L. R., "Application of Computational Aerodynamics to Airplane Design," *Journal of Aircraft*, Vol. 21, No. 6, 1984, pp. 355–370.

[3]Butter, D. J., Hunt, B., and Hargreaves, G. R., "A Survey of Boundary Integral Methods," *Numerical Methods in Aeronautical Fluid Dynamics*, Academic Press, London, 1982, pp. 471–515.

[4]Sudhakar, K., "Three Dimensional Ducts and Low Order Panel Methods," *Proceedings of the CAARC Specialists Meeting on CFD*, Dec. 5–10, 1988, National Aeronautical Laboratory, Bangalore, India.

[5]Guo, R. W., and Seddon, J., "The Swirl in an S-Duct of Typical Air Intake Proportions," *The Aeronautical Quarterly*, Vol. 34, May 1983, pp. 99–129.